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REPAIR OF A PLATE WITH A CIRCULAR HOLE BY APPLYING A PATCH

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The stressed state of a thin elastic infinite plate with a circular hole covered by a circular patch of a greater radius is considered. The center of the hole coincides with the center of the patch. The patch is attached to the plate along its entire boundary. Stresses are prescribed at infinity on the plate and at the hole boundary. Complex Muskhelishvili potentials are found by the method of power series, and the behavior of stresses on the patch–plate interface and at the hole boundary is studied.

Key words: *plate with a hole, complex Muskhelishvili potentials, stresses.*

1. Formulation of the Problem. Let a thin elastic plate S with a circular hole, which occupies the domain $|z| \geqslant R$ in the plane $z = x + iy$, be covered by a thin elastic circular patch S_1: $|z| \leqslant R_1$ ($R \leqslant R_1$) attached to the plate without tension and interlayers along the boundary L_1: $|z| = R_1$. The plate and the patch are uniform and isotropic and have the thicknesses, shear moduli, and Poisson's ratios h, μ, ν and h_1, μ_1, ν_1, respectively. Specified normal stresses σ_x^∞ and σ_y^∞, shear stress τ_{xy}^∞, and rotation ω^∞ act at infinity in the plane of the plate; specified normal stress σ_r and shear stress $\tau_{r\theta}$ act at the boundary L: $|z| = R$:

$$(\sigma_r + i\tau_{r\theta})(t) = p(t), \qquad t \in L. \tag{1.1}$$

The line L_1 of the patch–plate interface obeys the conditions of equal displacements of the points of this line relative to the plate and the patch and the condition of equilibrium of all points of this line:

$$\begin{gathered}(u + iv)_1(t) = (u + iv)_2(t) = (u + iv)_3(t),\\ h_1(\sigma_r + i\tau_{r\theta})_1(t) + h(\sigma_r + i\tau_{r\theta})_2(t) = h(\sigma_r + i\tau_{r\theta})_3(t), \qquad t \in L_1.\end{gathered} \tag{1.2}$$

Here $u+iv$ is the displacement vector; the patch is indicated by the subscript 1, the subscript 2 refers to the domain S_2: $R < |z| < R_1$ of the plate inside the interface line, and the subscript 3 refers to the domain S_3: $|z| > R_1$ of the plate outside the interface line.

We assume that the plate and patch surfaces touch each other without friction and are subjected to a generalized planar stressed state determined by the Kolosov–Muskhelishvili formulas [1] in polar coordinates:

$$\begin{gathered}(\sigma_r + \sigma_\theta)_k(z) = 4\,\mathrm{Re}\,\Phi_k(z),\\ (\sigma_r + i\tau_{r\theta})_k(z) = \Phi_k(z) + \overline{\Phi_k(z)} - \overline{z\Phi_k'(z)} - z^{-1}\overline{z\Psi_k(z)},\\ 2\mu_k \frac{\partial}{\partial\theta}(u + iv)_k(z) = iz(æ_k\Phi_k(z) - \overline{\Phi_k(z)} + \overline{z\Phi_k'(z)} + z^{-1}\overline{z\Psi_k(z)}\,),\\ 2\mu_k \frac{\partial}{\partial r}(u + iv)_k(z) = \mathrm{e}^{i\theta}(æ_k\Phi_k(z) - \overline{\Phi_k(z)}\,) - r\overline{\Phi_k'(z)} - \mathrm{e}^{-i\theta}\,\overline{\Psi_k(z)},\\ z \in S_k, \qquad æ_1 = \frac{3 - \nu_1}{1 + \nu_1}, \qquad æ_2 = æ_3 = æ = \frac{3 - \nu}{1 + \nu}, \qquad \mu_2 = \mu_3 = \mu, \qquad k = 1, 2, 3.\end{gathered} \tag{1.3}$$

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Here σ_r, σ_θ, and $\tau_{r\theta}$ are the components of the stress tensor in polar coordinates r, θ ($r\,\mathrm{e}^{i\theta} = x + iy$) per unit thickness of the plate or the patch; $\Phi_k(z)$ and $\Psi_k(z)$ are the single-valued analytical functions (complex potentials) in the domain S_k; at infinity, we have

$$\Phi_3(z) = \Gamma + Qz^{-1} + O(z^{-2}), \qquad \Psi_3(z) = \Gamma' - æ\overline{Q}z^{-1} + O(z^{-2}), \tag{1.4}$$

where

$$\Gamma = \frac{\sigma_x^\infty + \sigma_y^\infty}{4} + \frac{2i\mu}{1+æ}\,\omega^\infty, \qquad \Gamma' = \frac{\sigma_y^\infty - \sigma_x^\infty}{2} + i\tau_{xy}^\infty, \qquad Q = -\frac{X+iY}{2\pi(1+æ)h}.$$

In Eqs. (1.4), $X + iY = ih\int\limits_L p(t)\,dt = -hR\int\limits_0^{2\pi} p(R\,\mathrm{e}^{i\theta})\,\mathrm{e}^{i\theta}\,d\theta$ is the main vector of external forces acting at the hole boundary.

2. Solution of the Problem. We write the specified function $p(t)$ ($t \in L$) as a function of the polar angle θ:

$$p(t) = p(R\,\mathrm{e}^{i\theta}) = g(\theta), \qquad 0 \leqslant \theta \leqslant 2\pi \quad (R = \mathrm{const}). \tag{2.1}$$

As in solving the main problems of the elasticity theory for a circle [1], we assume that the function $g(\theta)$ is continuously differentiable in the interval $[0, 2\pi]$, satisfies the conditions $g(0) = g(2\pi)$ and $g'(0) = g'(2\pi)$, and has the second derivative that satisfies the Dirichlet condition. Then, this function can be expanded into the complex Fourier series

$$p(R\,\mathrm{e}^{i\theta}) = \sum_{n=-\infty}^{+\infty} A_n\,\mathrm{e}^{in\theta}, \qquad 0 \leqslant \theta \leqslant 2\pi, \tag{2.2}$$

where

$$A_n = \frac{1}{2\pi}\int\limits_0^{2\pi} p(R\,\mathrm{e}^{i\theta})\,\mathrm{e}^{-in\theta}\,d\theta, \qquad n = 0, \pm 1, \ldots\ .$$

The coefficients A_n satisfy the inequalities

$$|A_n| \leqslant M/|n|^3, \qquad M = \mathrm{const} > 0 \quad (n = \pm 1, \pm 2, \ldots), \tag{2.3}$$

which provide the validity of all subsequent operations performed with the power series.

We seek the complex potentials in the domains S_k in the form of the power series

$$\Phi_k(z) = \sum_{n=-\infty}^{+\infty} a_{nk} z^n, \qquad \Psi_k(z) = \sum_{n=-\infty}^{+\infty} b_{nk} z^n, \tag{2.4}$$

where

$$a_{n1} = b_{n1} = 0, \quad (n = -1, -2, \ldots), \qquad a_{n3} = b_{n3} = 0 \quad (n = 1, 2, \ldots) \tag{2.5}$$

and, according to (1.4),

$$a_{03} = \Gamma, \qquad a_{-13} = Q, \qquad b_{03} = \Gamma', \qquad b_{-13} = -æ\overline{Q}. \tag{2.6}$$

On the basis of conditions (1.1) and (1.2) and formulas (1.3), we have the following boundary conditions on the circumferences L and L_1:

$$\begin{gathered}
\Phi_2(t) + \overline{\Phi_2(t)} - \overline{t\Phi_2'(t)} - t^{-1}\overline{t\Psi_2(t)} = p(t), \qquad t = R\,\mathrm{e}^{i\theta}, \\
\mu_*^{-1}(æ_1\Phi_1(t) - \overline{\Phi_1(t)} + \overline{t\Phi_1'(t)} + t^{-1}\overline{t\Psi_1(t)}\,) = æ\Phi_2(t) - \overline{\Phi_2(t)} + \overline{t\Phi_2'(t)} + t^{-1}\overline{t\Psi_2(t)} \\
= æ\Phi_3(t) - \overline{\Phi_3(t)} + \overline{t\Phi_3'(t)} + t^{-1}\overline{t\Psi_3(t)}, \qquad t = R_1\,\mathrm{e}^{i\theta}, \\
h_*(\Phi_1(t) + \overline{\Phi_1(t)} - \overline{t\Phi_1'(t)} - t^{-1}\overline{t\Psi_1(t)}\,) + \Phi_2(t) + \overline{\Phi_2(t)} - \overline{t\Phi_2'(t)} - t^{-1}\overline{t\Psi_2(t)} \\
= \Phi_3(t) + \overline{\Phi_3(t)} - \overline{t\Phi_3'(t)} - t^{-1}\overline{t\Psi_3(t)}, \qquad t = R_1\,\mathrm{e}^{i\theta}, \\
0 \leqslant \theta \leqslant 2\pi, \qquad \mu_* = \mu_1/\mu, \qquad h_* = h_1/h.
\end{gathered} \tag{2.7}$$

Assuming that series (2.4) and the series obtained by term-by-term differentiation from $\Phi_k(z)$ to be uniformly converging in the corresponding domains S_k, including their boundaries, we substitute them into conditions (2.7). Then, taking into account (2.1), (2.2), (2.5), and (2.6), to find the remaining unknown coefficients a_{nk} and b_{nk} of these series, we obtain an infinite system of linear algebraic equations, which is decomposed into finite systems with respect to individual groups of coefficients. Their solutions are found by the formulas

$$a_{02} = \frac{(æ_1 - 1)(æ + 1)\operatorname{Re}\Gamma + 2h_*\mu_* R_*^2 \operatorname{Re} A_0}{2h_*\mu_*(æ - 1 + 2R_*^2) + (æ_1 - 1)(æ + 1)} + i\operatorname{Im}\Gamma, \qquad a_{12} = \frac{2(1 - R_*^2)\overline{Q} + R_1 R_*^3 A_1}{R_1^2 \beta_1},$$

$$a_{01} = \frac{\mu_*}{æ_1 - 1}\left((æ - 1 + 2R_*^2)\operatorname{Re} a_{02} - R_*^2 \operatorname{Re} A_0\right) + \frac{i\mu_*}{æ_1 + 1}\left((æ + 1)\operatorname{Im}\Gamma - R_*^2 \operatorname{Im} A_0\right),$$

$$a_{11} = -\frac{æ + 1}{h_*} a_{12}, \qquad a_{22} = \frac{\Delta_2}{R_1^2}\Big(3\,\frac{æ + 1}{\mu_*}(1 - R_*^2)\Gamma' + \gamma_2 R_*^4 A_2 + 3\alpha(1 - R_*^2)\overline{A_{-2}}\Big),$$

$$a_{21} = -(æ + 1)h_*^{-1} a_{22}, \qquad a_{-22} = R_1^2 \Delta_2\left(\mu_*^{-1}(æ + 1)\beta_2 \overline{\Gamma}' + \alpha\beta_2 A_{-2} - \alpha(1 - R_*^2)R_*^4 \overline{A_2}\,\right),$$

$$a_{-23} = \alpha^{-1}\mu_*^{-1}\left((æ + 1)a_{-22} - h_*\mu_* R_1^2 \overline{\Gamma}'\,\right), \qquad a_{-12} = Q, \qquad b_{-12} = -æ\overline{Q},$$

$$b_{01} = \mu_*\Gamma' + \mu_* æ R_1^{-2}\overline{a_{-23}} - R_1^2 a_{21}, \qquad b_{02} = R^{-2}\overline{a_{-22}} - R^2 a_{22} - \overline{A_{-2}},$$

$$b_{-33} = æ\overline{a_{12}} R_1^4 + b_{-32}, \qquad b_{-32} = \overline{a_{12}} R^4 + 2QR^2 - \overline{A_1} R^3, \tag{2.8}$$

$$b_{-23} = \mu_*^{-1} R_1^2 (æ_1 - 1)\operatorname{Re} a_{01} - R_1^2 (æ - 1)\operatorname{Re}\Gamma + iR^2 \operatorname{Im} A_0, \qquad b_{-22} = 2R^2 \operatorname{Re} a_{02} - R^2 \overline{A_0},$$

$$b_{-42} = 3R^2 a_{-22} + R^6 \overline{a_{22}} - R^4 \overline{A_2}, \qquad b_{-43} = \mu_*^{-1} æ_1 R_1^6 \overline{a_{21}} + 3R_1^2 a_{-23},$$

$$a_{n2} = R_1^{-n}\Delta_n\left(\gamma_n R_*^{n+2} A_n + (n + 1)\alpha(1 - R_*^2)R_*^{2-n}\overline{A_{-n}}\,\right), \qquad a_{n1} = -(æ + 1)h_*^{-1} a_{n2},$$

$$a_{-n2} = R_1^n \Delta_n \alpha\left(R_*^{2-n}\beta_n A_{-n} - (n - 1)(1 - R_*^2)R_*^{n+2}\overline{A_n}\,\right), \qquad a_{-n3} = \alpha^{-1}\mu_*^{-1}(æ + 1)a_{-n2},$$

$$b_{-(n+2)2} = (n + 1)R^2 a_{-n2} + R^{2n+2}\overline{a_{n2}} - R^{n+2}\overline{A_n}, \quad b_{-(n+2)3} = \mu_*^{-1} æ_1 R_1^{2n+2}\overline{a_{n1}} + (n + 1)R_1^2 a_{-n3},$$

$$b_{(n-2)1} = \mu_* æ R_1^{2-2n}\overline{a_{-n3}} - (n - 1)R_1^2 a_{n1}, \qquad b_{(n-2)2} = R^{2-2n}\overline{a_{-n2}} - (n - 1)R^2 a_{n2} - \overline{A_{-n}} R^{2-n},$$

$$b_{(n-2)1} = \frac{\mu_* æ \overline{a_{-n3}}}{R_1^{2n-2}} - (n - 1)R_1^2 a_{n1}, \quad b_{(n-2)2} = \frac{\overline{a_{-n2}}}{R^{2n-2}} - \frac{\overline{A_{-n}}}{R^{n-2}} - (n - 1)R^2 a_{n2}, \quad n = 3, 4, \ldots,$$

where $R_* = R/R_1$, $\alpha = \mu_*^{-1}(æ + 1 + h_*\mu_* æ)$, $\beta_n = æ + \mu_*^{-1} h_*^{-1} æ_1 (æ + 1) + R_*^{2n+2}$, $\gamma_n = h_* æ^2 + R_*^{2-2n}\alpha$, and $\Delta_n = (\beta_n \gamma_n + \alpha(n^2 - 1)(1 - R_*^2)^2)^{-1}$.

It follows from these formulas and inequalities (2.3) that, as $n \to \infty$, the coefficients of series (2.4) decrease as $|n|^{-3}$, which ensures the convergence of these series.

Remark 1. The solution of the problem in the case of sealing a circular hole by a patch of the same radius can be obtained from the resultant solution directly by the limiting transition $R_1 \to R$. The thus-obtained formulas coincide, with accuracy to notation, with the formulas in [2]. The conjugation conditions (1.2) can be written as

$$(u + iv)_1(t) = (u + iv)_3(t), \quad h_1(\sigma_r + i\tau_{r\theta})_1(t) = h(\sigma_r + i\tau_{r\theta})_3(t), \quad (u + iv)_2(t) = (u + iv)_1(t) \quad (t \in L)$$

(the latter condition does not affect the final solution of the problem).

3. Investigation of the Stressed State for $p(t) = 0$. If the hole boundary is free from stresses, all coefficients $A_n = 0$. Then, from formulas (2.4) and (2.8), we obtain the following representations for the complex potentials:

$$\Phi_k(z) = a_{-2k} z^{-2} + a_{0k} + a_{2k} z^2, \tag{3.1}$$

$$\Psi_k(z) = b_{-4k} z^{-4} + b_{-2k} z^{-2} + b_{0k}, \qquad z \in S_k \quad (k = 1, 2, 3)$$

($a_{-21} = a_{23} = b_{-41} = b_{-21} = 0$). Then, the stresses at the point $z = r\,e^{i\theta} \in S_k$ of the plate or the patch, according to (1.3), are found by the formulas

$$
\begin{aligned}
\sigma_r(z)_k &= 2\operatorname{Re} a_{0k} - r^{-2}\operatorname{Re} b_{-2k} + (4r^{-2}\operatorname{Re} a_{-2k} - r^{-4}\operatorname{Re} b_{-4k} - \operatorname{Re} b_{0k})\cos 2\theta \\
&\quad + (4r^{-2}\operatorname{Im} a_{-2k} - r^{-4}\operatorname{Im} b_{-4k} + \operatorname{Im} b_{0k})\sin 2\theta, \\
\sigma_\theta(z)_k &= 2\operatorname{Re} a_{0k} + r^{-2}\operatorname{Re} b_{-2k} + (4r^{2}\operatorname{Re} a_{2k} + r^{-4}\operatorname{Re} b_{-4k} + \operatorname{Re} b_{0k})\cos 2\theta \\
&\quad + (-4r^{2}\operatorname{Im} a_{2k} + r^{-4}\operatorname{Im} b_{-4k} - \operatorname{Im} b_{0k})\sin 2\theta, \\
\tau_{r\theta}(z)_k &= r^{-2}\operatorname{Im} b_{-2k} + (2r^{2}\operatorname{Im} a_{2k} - 2r^{-2}\operatorname{Im} a_{-2k} + r^{-4}\operatorname{Im} b_{-4k} + \operatorname{Im} b_{0k})\cos 2\theta \\
&\quad + (2r^{2}\operatorname{Re} a_{2k} + 2r^{-2}\operatorname{Re} a_{-2k} - r^{-4}\operatorname{Re} b_{-4k} + \operatorname{Re} b_{0k})\sin 2\theta, \qquad k = 1, 2.
\end{aligned} \tag{3.2}
$$

We determine for which values of the polar angle θ the stresses σ_r, σ_θ, and $\tau_{r\theta}$ on the circumference $|z| = r$ reach their extreme values. For this purpose, omitting the arguments and subscripts at stresses for convenience, we write Eqs. (3.2) in the form

$$\sigma_r = \alpha_1 + \operatorname{Re}(c_1 e^{2i\theta}), \qquad \sigma_\theta = \alpha_2 + \operatorname{Re}(c_2 e^{2i\theta}), \qquad \tau_{r\theta} = \alpha_3 + \operatorname{Im}(c_3 e^{2i\theta}),$$

where α_1, α_2, and α_3 are the real coefficients independent of the polar angle θ; $c_1 = 4r^{-2}\overline{a_{-2k}} - r^{-4}\overline{b_{-4k}} - b_{0k}$, $c_2 = 4r^2 a_{2k} + r^{-4}\overline{b_{-4k}} + b_{0k}$, and $c_3 = 2r^2 a_{2k} + 2r^{-2}\overline{a_{-2k}} - r^{-4}\overline{b_{-4k}} + b_{0k}$. Since $a_{-21} = a_{23} = b_{-41} = 0$, and the remaining unknown constants a_{2k}, $\overline{a_{-2k}}$, b_{0k}, and $\overline{b_{-4k}}$, as is seen from Eqs. (2.8), are directly proportional to the number Γ' with real proportionality coefficients, we have $c_j = d_j\Gamma'$, where d_j are some real numbers. Hence, $\sigma_r = \alpha_1 + d_1|\Gamma'|\operatorname{Re} e^{i(\arg\Gamma' + 2\theta)}$, $\sigma_\theta = \alpha_2 + d_2|\Gamma'|\operatorname{Re} e^{i(\arg\Gamma' + 2\theta)}$, and $\tau_{r\theta} = \alpha_3 + d_3|\Gamma'|\operatorname{Im} e^{i(\arg\Gamma' + 2\theta)}$; the stresses σ_r and σ_θ $|z| = r$ reach their extreme values on the circumference $|z| = r$ at polar angles $\theta_1 = -\arg\Gamma'/2$ and $\theta_2 = (\pi - \arg\Gamma')/2$, whereas the stress $\tau_{r\theta}$ reaches its extreme value at $\theta_3 = (\pi - 2\arg\Gamma')/4$ and $\theta_4 = -(\pi + 2\arg\Gamma')/4$.

Thus, on each circumference $|z| = r$, the extreme values of stresses are reached at point that have the same polar angles θ_1 and θ_2 or θ_3 and θ_4, which depend neither on the polar radius of these points nor on elastic and geometric parameters of the plate and the patch, but depend only on $\arg\Gamma'$, i.e., on the force parameters acting at infinity.

To find the extreme values of stresses in each domain S_k, we have to assume that $\theta = \theta_1$ and $\theta = \theta_2$ or $\theta = \theta_3$ and $\theta = \theta_4$ in Eqs. (3.2) and find the extreme values of the resultant power functions of the polar radius r varied in the range determined by the chosen domain S_k.

To find the displacements of the points of the lines L and L_1, we use the formulas

$$(u + iv)(R_1 e^{i\theta}) = \int_0^{R_1} \frac{\partial(u + iv)_1}{\partial r}\, dr + \int_0^{\theta} \frac{\partial(u + iv)_1}{\partial \theta}\, d\theta,$$

$$(u + iv)(R e^{i\theta}) = \int_0^{R_1} \frac{\partial(u + iv)_1}{\partial r}\, dr + \int_{R_1}^{R} \frac{\partial(u + iv)_2}{\partial r}\, dr + \int_0^{\theta} \frac{\partial(u + iv)_2}{\partial \theta}\, d\theta,$$

and also formulas (1.3), (3.1). After simple transformations, we obtain the equalities

$$
\begin{aligned}
(u + iv)(R_1 e^{i\theta}) &= R_1(-(\overline{a_{21}}R_1^2 + \overline{b_{01}})e^{-i\theta} + (æ_1 a_{01} - \overline{a_{01}})e^{i\theta} + æ_1 a_{21}R_1^2 e^{3i\theta}/3)/(2\mu_1), \\
(u + iv)(R e^{i\theta}) &= R(-(æ a_{-22}R^{-2} + \overline{a_{22}}R^2 + \overline{b_{02}})e^{-i\theta} + (æ a_{02} - \overline{a_{02}} + \overline{b_{-22}}R^{-2})e^{i\theta} \\
&\quad + (æ a_{22}R^2 - 3\overline{a_{-22}}R^{-2} + \overline{b_{-42}}R^{-4})e^{3i\theta}/3)/(2\mu).
\end{aligned}
$$

Examples. Let the plate and the patch whose thickness is half of the plate thickness have elastic constants $\mu = 40$ MPa and $\nu = 0.37$ (for Cu) and $\mu_1 = 174.2$ MPa and $\nu_1 = 0.22$ (for Al_2O_3 alloy). The ratio of the hole and patch radii is 1 : 2. At infinity, the plate is subjected only to the tensile stress $\sigma_x^\infty = \sigma$ MPa or only the shear stress $\tau_{xy}^\infty = \sigma$ MPa (per unit thickness of the plate). All the remaining initial force parameters are zero.

The solid curves in Fig. 1 show the deformation of the hole boundaries L and the patch boundaries L_1; the dashed curves show the initial positions of the circumferences L and L_1 before application of the loads. To be more illustrative, the displacements of the points of the lines L and L_1 are taken with the coefficient $\mu/(2\sigma R_1)$. The cases with $\sigma_x^\infty \neq 0$ and $\tau_{xy}^\infty \neq 0$ are depicted in Fig. 1a and Fig. 1b, respectively.

If only the load $\sigma_x^\infty = \sigma$ is applied to the plate, the stresses σ_r (Fig. 2a), $\tau_{r\theta}$ (Fig. 2b), and σ_θ (Fig. 2c) on the upper half of the interface line L_1 on the side of both the plate and the patch are plotted in Fig. 2 as functions of the polar angle θ ($0 \leqslant \theta \leqslant \pi$). The stresses on the lower half of the line L_1 ($-\pi \leqslant \theta \leqslant 0$) are distributed symmetrically. The following notation is used here and in the subsequent figures: stresses on L_1 from the side of

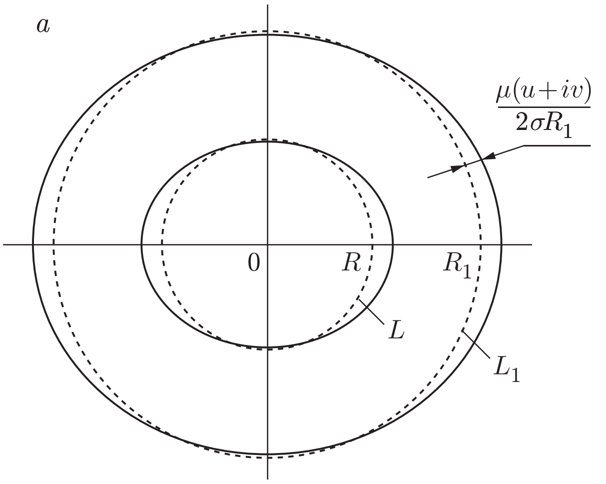
a
b
$\frac{\mu(u+iv)}{2\sigma R_1}$
0
R
R_1
L
L_1

Fig. 1

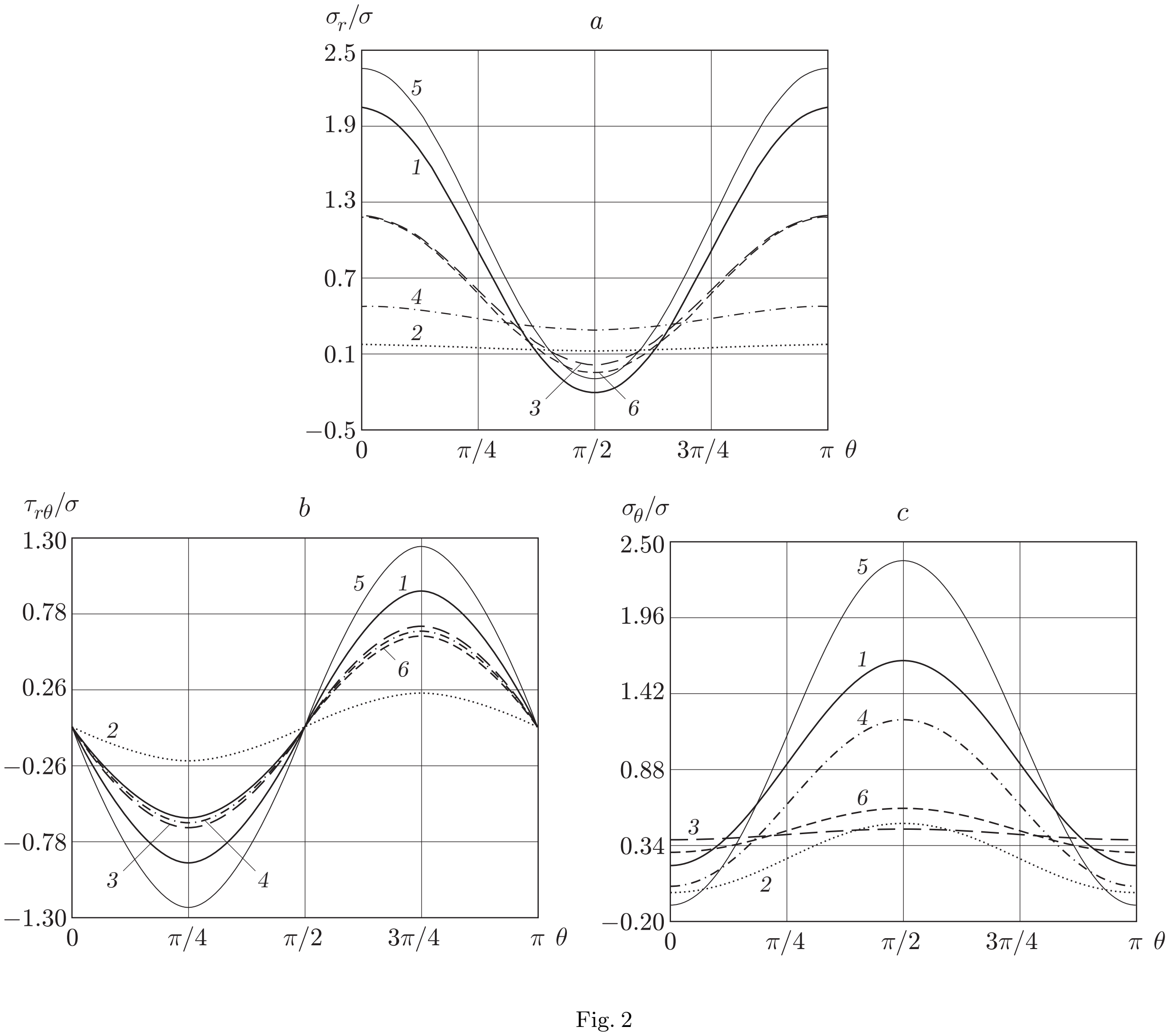
σ_r/σ
a
2.5
1.9
1.3
0.7
0.1
−0.5
0
π/4
π/2
3π/4
π θ
$\tau_{r\theta}/\sigma$
b
1.30
0.78
0.26
−0.26
−0.78
−1.30
σ_θ/σ
c
2.50
1.96
1.42
0.88
0.34
−0.20
1
2
3
4
5
6

Fig. 2

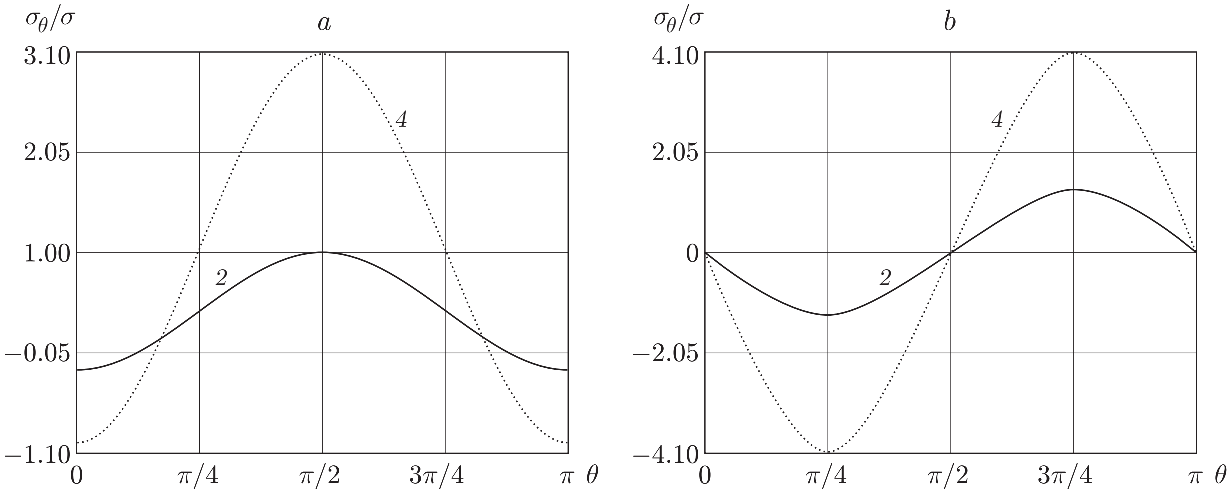

Fig. 3

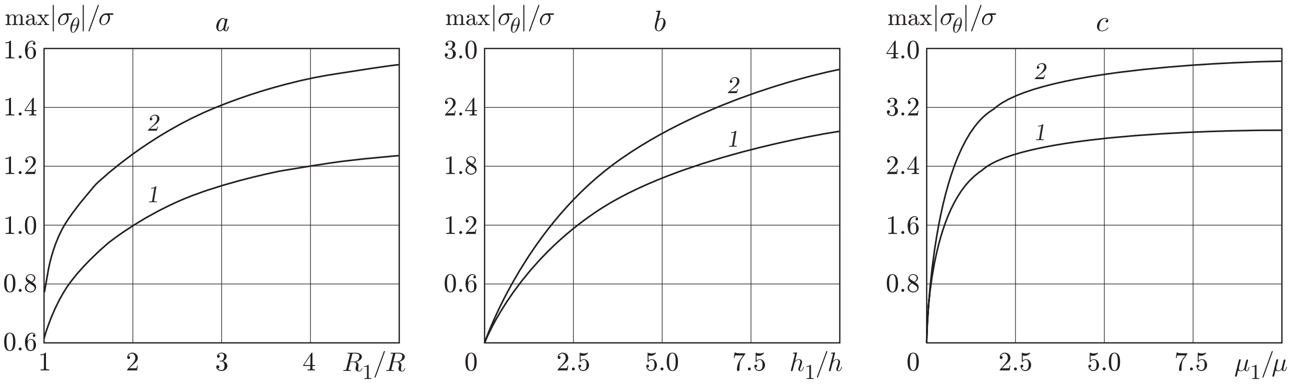

Fig. 4

the patch (1), stresses on the inner (2) and outer (3) sides of the line L_1 from the side of the plate, stresses in the case of the classical problem of extension of a plate with a stress-free hole $|z| \leqslant R$ under the action of a remote load σ_x^∞ (4), stresses on L_1 from inside (5) and outside (6) in the case of sealing a circular hole of radius R_1 by a patch of the same radius.

As it follows from Fig. 2, the presence of a circular ring S_2 in the problem considered decreases the concentration of stresses on the interface line L_1 from the side of the patch (curve 1) as compared to sealing of the hole of radius R_1 by a patch of the same radius (curve 5), i.e., S_2 plays the role of a stringer.

Figure 3 shows the tensile stress σ_θ at the hole boundary L for $\sigma_x^\infty = \sigma$ (Fig. 3a) and $\tau_{xy}^\infty = \sigma$ (Fig. 3b). The solid curve shows the plots of σ_θ at the hole boundary reinforced by a patch; the dotted curve shows the data with the patch absent. The stresses σ_r and $\tau_{r\theta}$ on L equal zero *a priori*. As it follows from Fig. 3, in the example considered, the presence of a patch decreases the concentration of the stress σ_θ at the hole boundary severalfold.

Figure 4 shows the maximum absolute values of the tensile stress σ_θ in the plate at the hole boundary reinforced by a patch versus the ratios of the patch-to-plate radii R_1/R (Fig. 4a), thicknesses h/h_1 (Fig. 4b), and shear moduli μ/μ_1 (Fig. 4c). Curves 1 and 2 refer to plate loading by the stresses $\sigma_x^\infty = \sigma$ and $\tau_{xy}^\infty = \sigma$, respectively. If there is no patch, $\max|\sigma_\theta|$ at the hole boundary equals 3σ for $\sigma_x^\infty = \sigma$ and 4σ for $\tau_{xy}^\infty = \sigma$. The maximum value of $|\sigma_\theta|$ is reached at the points with the polar angles $\theta = \pm\pi/2$ for $\sigma_x^\infty = \sigma$ and at the points with the polar angles $\theta = \pm\pi/4$ and $\theta = \pm 3\pi/4$ for $\tau_{xy}^\infty = \sigma$.

In other formulations, the problem of repair of plates with defects by means of patches is considered in [3–5].

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